1.

(i) Given that $y = \ln\left(\dfrac{1 + \sin 4x}{\cos 4x}\right)$, show that $\dfrac{dy}{dx} = \dfrac{4}{\cos 4x}$. [4]

(ii) Find $\displaystyle\int\left(\dfrac{\cos 2x}{\cos 2x + \sin 2x} + \dfrac{\sin 2x}{\cos 2x - \sin 2x}\right)dx$. [4]

2.

(a) Differentiate the following with respect to x.

(i) $\dfrac{1}{(3x - 4)^2}$ [1]

(ii) $\dfrac{\ln(x + 2)}{x}$ [3]

(b) Find $\displaystyle\int e^{(2x+3)}dx$. [3]

END OF QUESTION paper

Mark scheme

Question				Answer/Indicative content	Marks	Guidance	
				$\dfrac{\cos 4x \times 4\cos 4x - (1+\sin 4x) \times -4\sin 4x}{\cos^2 4x}$	M1	quotient rule; allow sign errors and / or one coefficient error	
				$\dfrac{4\cos^2 4x + 4\sin^2 4x + 4\sin 4x}{\cos^2 4x}$ oe	A1		
				$\dfrac{\cos 4x}{1+\sin 4x} \times their \dfrac{4(1+\sin 4x)}{\cos^2 4x}$	M1		
				$= \dfrac{4}{\cos 4x}$ **NB AG**	A1	use of chain rule; may be unsimplified	
				alternatively			
				$\dfrac{4\cos 4x}{1+\sin 4x} - \dfrac{-4\sin 4x}{\cos 4x}$	M1		
1		i		$\dfrac{4\cos 4x \times \cos 4x + 4\sin 4x(1+\sin 4x)}{(1+\sin 4x)\cos 4x}$	M1	chain rule; allow sign errors and / or one error in coefficient of $\cos 4x$ or $\sin 4x$	or use of product rule with $(1+\sin 4x)$ and $(\cos 4x)-1$ or $\sec 4x$
				eg $\dfrac{4(\cos^2 4x + \sin^2 4x) + 4\sin 4x}{(1+\sin 4x)\cos 4x}$	A1		
				$\dfrac{4}{\cos 4x}$	A1	combine to a single fraction FT *their* chain rule	$(1+\sin 4x) \times -1(\cos 4x)^{-2} \times -4\sin 4x$
				alternatively			
				$\dfrac{1}{\sec 4x + \tan 4x} \times \left(4\sec 4x\tan 4x + 4\sec^2 4x\right)$	M1	any equivalent correct step	$+\dfrac{4\cos 4x}{\cos 4x}$
					M1		
				$\dfrac{4\sec 4x(\tan 4x + \sec 4x)}{\sec 4x + \tan 4x}$	A1		
				$4\sec 4x$	A1		

		$\dfrac{4}{\cos 4x}$	[4]	allow sign errors and / or one coefficient error factorising – allow one coefficient slip	

A variety of approaches were seen. Most went straight to the quotient rule and the chain rule and went on to derive the given result successfully. Almost as many separated the logarithms and used the chain rule before combining to a single fraction, and they were equally successful. A few converted to the reciprocal forms and were generally successful. A significant minority made slips with coefficients, signs and brackets, thus losing the accuracy marks. Candidates are reminded of the need for rigour when deriving a given result.

	ii	$\dfrac{\cos 2x(\cos 2x - \sin 2x) + \sin 2x(\cos 2x + \sin 2x)}{(\cos 2x + \sin 2x)(\cos 2x - \sin 2x)}$ oe	M1	combine into a single fraction; allow sign errors	allow equivalent form with double angle formulae
		$\dfrac{\cos^2 2x - \cos 2x \sin 2x + \sin 2x \cos 2x + \sin^2 2x}{(\cos^2 2x - \sin^2 2x)}$	A1		allow equivalent separate fractions with correct common denominator
		$\dfrac{1}{\cos 4x}$	A1	or better	

		$\dfrac{1}{4}\ln\left(\dfrac{1+\sin 4x}{\cos 4x}\right)+c$ oe eg $\dfrac{1}{4}\ln\left(1+\sin 4x\right)+\dfrac{1}{4}\ln\sec 4x+c$	A1 [4]	NB $\dfrac{1}{4}$ in (sec $4x$ + tan $4x$) + c	

Examiner's Comments

Candidates who failed to combine the integrand into a single fraction generally made no progress.

Of the good number who did adopt the correct strategy, a significant proportion made sign or coefficient errors and so were unable to make the connection with part (i).

		Total		8	

2	a	(i)	$-\dfrac{6}{(3x-4)^3}$ or $-6(3x-4)^{-3}$ oe	B1 (AO1.1) [1]	Allow M1 for denominator and one term in numerator correct oe two–layered fraction or fractions	
		(ii)	$\dfrac{x\times\dfrac{1}{x+2}-\ln(x+2)}{x^2}$ $=\dfrac{1}{x(x+2)}-\dfrac{\ln(x+2)}{x^2}$	M1 (AO1.1a) M1 (AO1.1) A1 (AO1.1) [2]		
	b		$\dfrac{1}{2}e^{(2x+3)}+c$	B1 (AO1.1a) B1 (AO1.1) B1	B1 for $e^{(2x+3)}$ B1 for $\dfrac{1}{2}$	

				(AO1.1) [2]	B1 for $+ c$	
			Total	7		